

CHAPTER TWO

CHANGE OF SUBJECT

Introduction:

To make a letter the subject of a given equation is to let it stand alone, on one side of the equal to symbol.

Q1. Given that $c = 2\pi r$,

- i) make r the subject.
- ii) calculate r when $c = 20$ and $\pi = 3.14$.

Soln.

$$c = 2\pi r$$

$$\text{Divide through using } 2\pi \Rightarrow \frac{c}{2\pi} = \frac{2\pi r}{2\pi} \Rightarrow \frac{c}{2\pi} = r \Rightarrow r = \frac{c}{2\pi}$$

When $c = 20$ and $\pi = 3.14$

$$\Rightarrow r = \frac{c}{2\pi} = \frac{20}{2(3.14)} = \frac{20}{6.28}$$

$$= 3.2 \Rightarrow r = 3.2.$$

Q2. Given that $M = RVL$,

- i) make V the subject.
- ii) Calculate V when $M = 50$, $R = 20$ and $L = 10$.

Soln.

$$i) \quad M = RVL$$

$$\text{Divide through using } RL \Rightarrow \frac{M}{RL} = \frac{RVL}{RL}, \Rightarrow \frac{M}{RL} = V \Rightarrow V = \frac{M}{RL}$$

$$ii) \quad \text{When } M = 50, R = 20 \text{ and } L = 10 \Rightarrow V = \frac{M}{RL} = \frac{50}{(20)(10)}$$

$$= \frac{50}{200} = 0.25.$$

Q3. You are given the formula $2RV^2 = mg$.

- i) Make R the subject.

- ii) Calculate R when $V = 3$, $M = 5$ and $g = 2$.

Soln.

i) $2RV^2 = mg$.

Divide through using $2V^2 \Rightarrow \frac{2RV^2}{2V^2} = R = \frac{mg}{2V^2}$

- ii) If $V = 3$, $m = 5$ and $g = 2$

$$\Rightarrow R = \frac{Mg}{2V^2} = \frac{(5)(2)}{(2)(3)^2} = \frac{10}{(2)(9)} = \frac{10}{18} \Rightarrow R = 0.55.$$

Q4. Given that $a + b = 2R$,

- make a the subject.
- calculate a, when $b = 3$ and $R = 10$.
- make R the subject.
- calculate R when $a = 3$ and $b = 5$.

Soln.

i) $a + b = 2R \Rightarrow a = 2R - b$

ii) When $b = 3$ and $R = 10 \Rightarrow a = 2R - b, \Rightarrow a = 2(10) - 3$
 $= 20 - 3 = 17.$

iii) To make R the subject, $a + b = 2R$. Divide through using 2 $\Rightarrow \frac{a+b}{2} = \frac{2R}{2} \Rightarrow \frac{a+b}{2} = R$
 $\Rightarrow R = \frac{a+b}{2}.$

When $a = 3$ and $b = 5$

$$\Rightarrow R = \frac{a+b}{2} = \frac{3+5}{2} = \frac{8}{2} = 4.$$

Q5. Given that $2V + 3R = 4b$,

- make V the subject.
- calculate V when $R = 3$ and $b = 1$.

Soln.

i) $2V + 3R = 4b \Rightarrow 2V = 4b - 3R$

Divide through using 2 $\Rightarrow \frac{2v}{2} = \frac{4b-3R}{2} \Rightarrow V = \frac{4b-3R}{2}$

- ii) When $R = 3$ and $b = 1$

$$\Rightarrow V = \frac{4(1) - 3(3)}{2} = \frac{4 - 9}{2} = \frac{-5}{2} = -2.5.$$

Q6. Given that the three quantities V, u and t are connected by the formula $V = 3u + at^2$, calculate a when $V = 10$, $t = 1$ and $u = 3$.

N/B: Before you can calculate a, you must first make a the subject.

Soln.

$$V = 3u + at^2 \Rightarrow V - 3u = at^2$$

$$\text{Divide through using } t^2 \Rightarrow \frac{V-3u}{t^2} = \frac{at^2}{t^2} \Rightarrow \frac{V-3u}{t^2} = a$$

$$\Rightarrow a = \frac{V-3u}{t^2}.$$

When $V = 10$, $t = 1$ and $u = 3$

$$\Rightarrow a = \frac{V-3u}{t^2} = \frac{10-3(3)}{1^2} = \frac{10-9}{1} = \frac{1}{1}$$

$$= 1.$$

$$\text{N/B: If } a^2 = 4 \Rightarrow a = \sqrt{4} = 2.$$

$$\text{If } x^2 = 25 \Rightarrow x = \sqrt{25} = 5.$$

$$\text{Q7. If } 2V = u - ga^2,$$

(i) make a the subject.

(ii) calculate a when $V = 3$, $u = 30$ and $g = 1$.

Soln.

$$(i) 2V = u - ga^2 \Rightarrow 2V + ga^2 = u$$

$$\Rightarrow ga^2 = u - 2V.$$

Divide through using g

$$\Rightarrow \frac{ga^2}{g} = \frac{u-2v}{g} \Rightarrow a^2 = \frac{u-2V}{g} \Rightarrow a = \sqrt{\frac{u-2v}{g}}.$$

(ii) When $V = 3$, $u = 30$ and $g = 1$

$$\Rightarrow a = \sqrt{\frac{30-2(3)}{1}} \Rightarrow a = \sqrt{\frac{30-6}{1}}$$

$$\Rightarrow a = \sqrt{\frac{24}{1}} \Rightarrow a = \sqrt{24} = 4.9.$$

Q8. The movement of a particle is such that its final velocity V , its initial velocity u , its acceleration a and its time t , are connected by the formula $V = 3u + at^2$. Calculate the time in seconds when $V = 115m/s$, $u = 5m/s$ and $a = 4m/s^2$.

Soln.

$$V = 3u + at^2 \Rightarrow V - 3u = at^2$$

$$\text{Divide through using } a \Rightarrow \frac{V-3u}{a} = \frac{at^2}{a}, \Rightarrow \frac{V-3u}{a} = t^2,$$

$$\Rightarrow t^2 = \frac{V-3u}{a} \Rightarrow t = \sqrt{\frac{V-3u}{a}}.$$

But $V = 115$, $u = 5$ and $a = 4$

$$\Rightarrow t = \sqrt{\frac{115 - 3(5)}{4}} = \sqrt{\frac{100}{4}} = \sqrt{25}$$

$$= 5 \Rightarrow t = 5 \text{ seconds.}$$

N/B: If the letter we are required to make the subject appears twice, then we must factorize it by bringing it outside the bracket.

Q9. The variables b and v are connected by the formula $5b - v^2 = vb + 2$. Make b the subject.

Soln.

$$5b - v^2 = vb + 2 \Rightarrow 5b = vb + 2 + v^2, \Rightarrow 5b - vb = 2 + v^2. \text{ Factorize the } b$$

$$\Rightarrow b(5 - v) = 2 + v^2$$

$$\text{Divide through using } 5 - v \Rightarrow \frac{b(5 - v)}{5 - v} = \frac{2 + v^2}{5 - v}$$

$$\Rightarrow b = \frac{2 + v^2}{5 - v}.$$

Q10. Given that $av^2 + 2 = 2v^2 - 2c$, make v the subject.

Soln.

$$av^2 + 2 = 2v^2 - 2c$$

$$\Rightarrow av^2 + 2 - 2v^2 = -2c,$$

$$\Rightarrow av^2 - 2v^2 = -2c - 2$$

$$\Rightarrow v^2(a - 2) = -2c - 2,$$

Divide through using $a - 2$.

$$\Rightarrow \frac{v^2(a - 2)}{(a - 2)} = \frac{-2c - 2}{(a - 2)}$$

$$\Rightarrow v^2 = \frac{-2c - 2}{(a - 2)} \Rightarrow v = \sqrt{\frac{-2c - 2}{a - 2}}.$$

N/B: (1) If $a^2 = b \Rightarrow a = \sqrt{b}$

(2) If $a^3 = b \Rightarrow a = \sqrt[3]{b}$

(3) If $a^4 = b \Rightarrow a = \sqrt[4]{b}$

(4) If $a^5 = b \Rightarrow a = \sqrt[5]{b}$.

Q11. Given that $5b^3 - 1 = v^2$

(i) make b the subject

(ii) calculate b when $v = 10$.

Soln.

$$5b^3 - 1 = v^2 \Rightarrow 5b^3 = v^2 + 1,$$

$$\Rightarrow \frac{5b^3}{5} = \frac{v^2 + 1}{5} \Rightarrow b^3 = \frac{v^2 + 1}{5},$$

$$\Rightarrow b = \sqrt[3]{\frac{v^2 + 1}{5}}. \quad \text{If } v = 10$$

$$\Rightarrow b = \sqrt[3]{\frac{10^2 + 1}{5}} = \sqrt[3]{\frac{100 + 1}{5}} = \sqrt[3]{\frac{101}{5}}$$

$$= \sqrt[3]{50.5} = 3.6.$$

Q12. If $b^3 + 3 = v^4 + 5$, make b the subject.

Soln.

$$b^3 + 3 = v^4 + 5 \Rightarrow b^3 = v^4 + 5 - 3, \Rightarrow b^3 = v^4 + 2 \Rightarrow b = \sqrt[3]{v^4 + 2}.$$

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